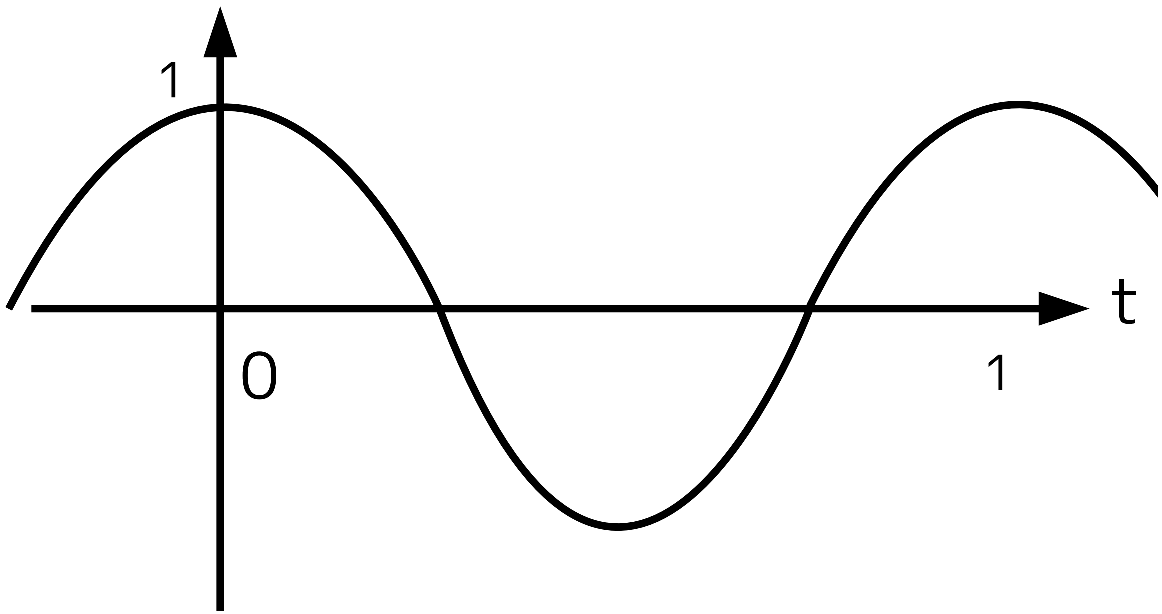


$$f(t) = \cos(2\pi t)$$

デルタ関数δ(t)の定義

$$\delta(x)=\begin{cases}0 & (if\, t\neq 0)\\ \infty & (if\, t=0)\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)\,dt=1$$

$$\int_{-\infty}^{\infty}\mathrm{e}^{-i2\pi f t}\,dt=\delta(f)$$

$$\int_{-\infty}^{\infty}\mathrm{e}^{i2\pi f t}\,df=\delta(t)$$

$$\int_{-\infty}^{\infty}\mathrm{x}(t)\delta(t-a)\,dt=x(a)$$

$$\int_{-\infty}^{\infty}\delta(i2\pi f t)\,dt=\int_{-\infty}^{\infty}\delta(i2\pi f t)e^{-i2\pi f t}\,dt=1$$

$$f(t)=\cos(2\pi t)$$

$$F(f)=\int_{-\infty}^{\infty}\cos(2\pi t)\ \ e^{-i2\pi f t}dt$$

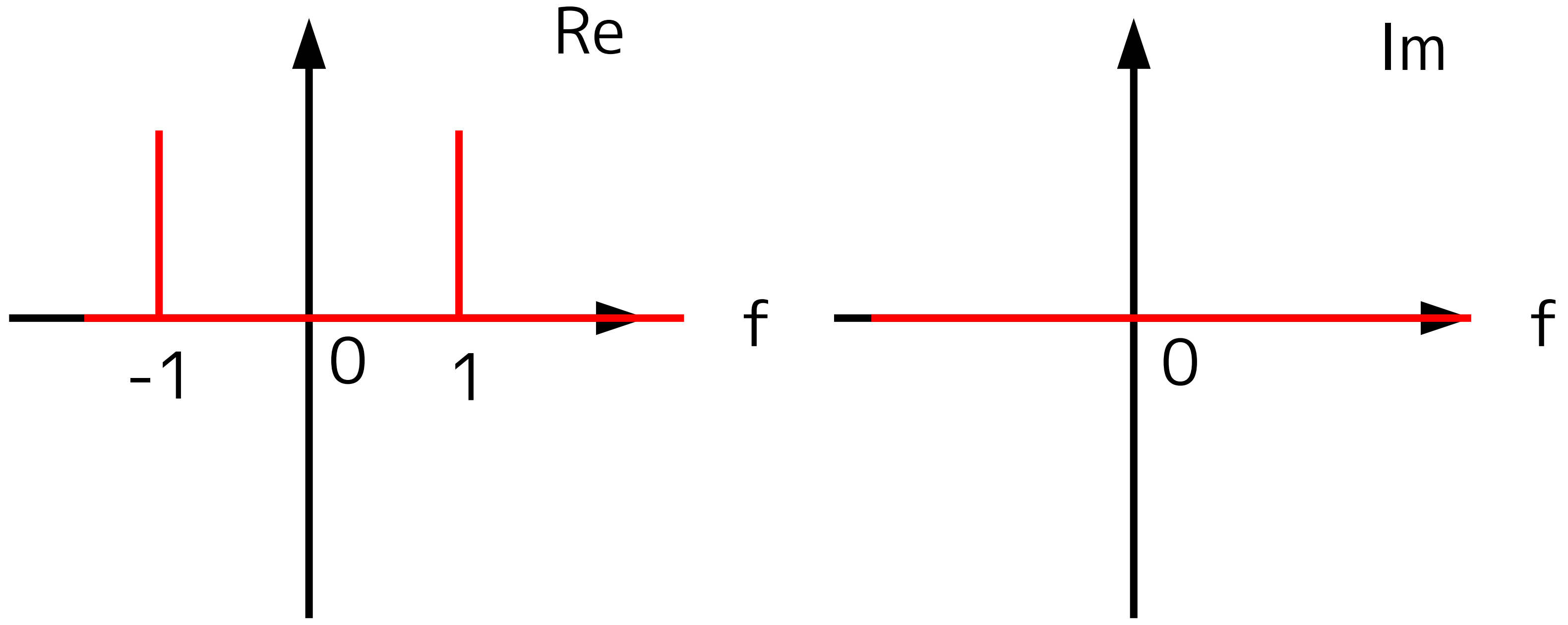
$$=\int_{-\infty}^{\infty}(\frac{e^{i2\pi t}+e^{-i2\pi t}}{2})e^{-i2\pi f t}dt$$

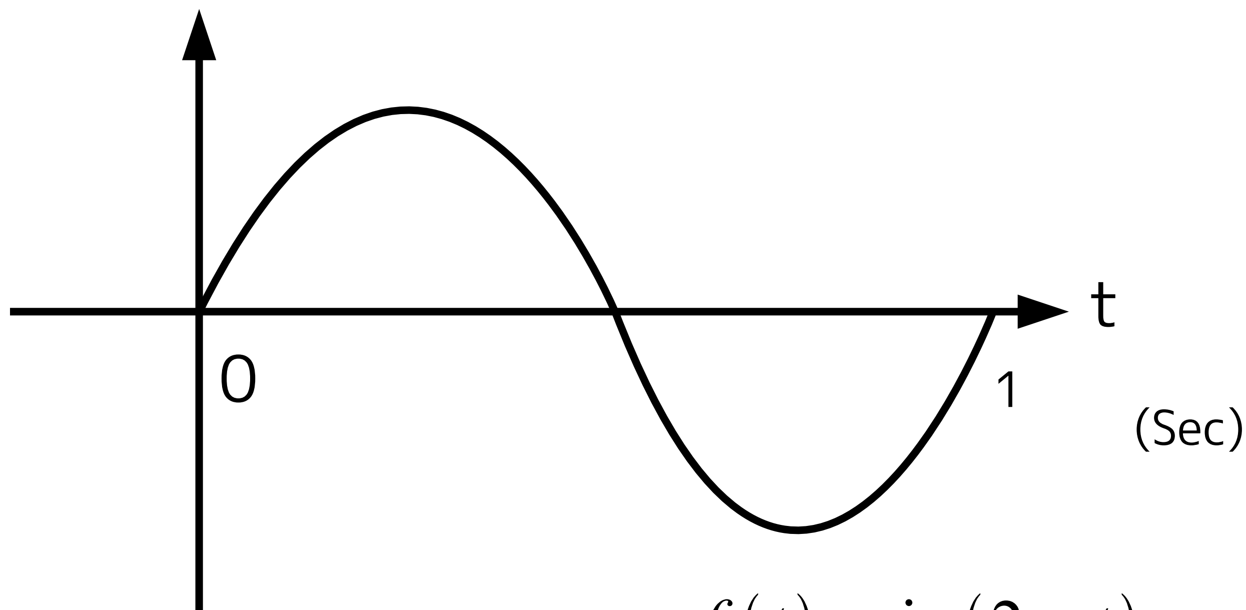
$$=\frac{1}{2}\int_{-\infty}^{\infty}(e^{-i(2\pi f-2\pi)t}+e^{-i(2\pi f+2\pi)t})dt$$

$$=\frac{1}{2}\left\{\int_{-\infty}^{\infty}e^{-i(2\pi f-2\pi)t}dt+\int_{-\infty}^{\infty}e^{-i(2\pi f+2\pi)t}dt\right\}$$

$$=\frac{1}{2}\left\{\delta(f-1)+\delta(f+1)\right\}$$

フーリエ変換の結果





$$f(t) = \sin(2\pi t)$$

$$f(t)=\sin 2\pi t$$

$$F(f)=\int_{-\infty}^{\infty}\sin(2\pi t)\quad e^{-i2\pi f}dt$$

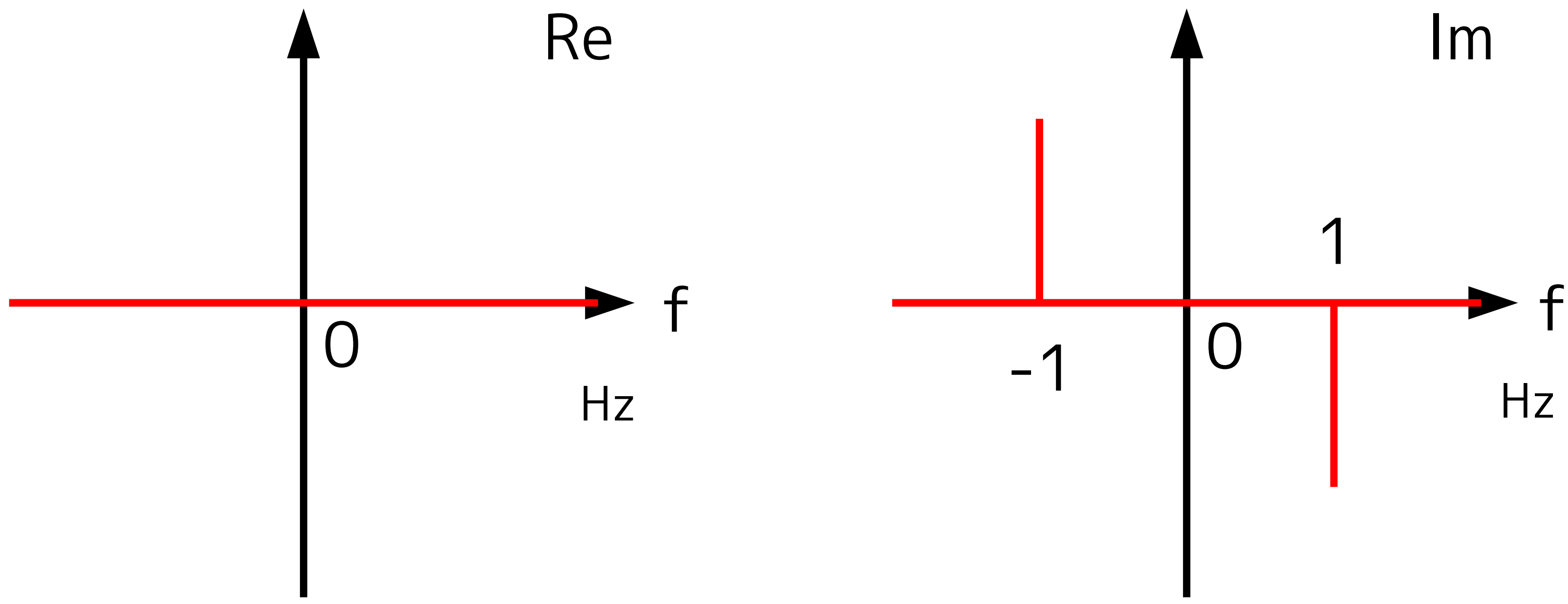
$$=\int_{-\infty}^{\infty}\left(\frac{e^{i2\pi t}-e^{-i2\pi t}}{2\mathrm{i}}\right)e^{-i2\pi f}dt$$

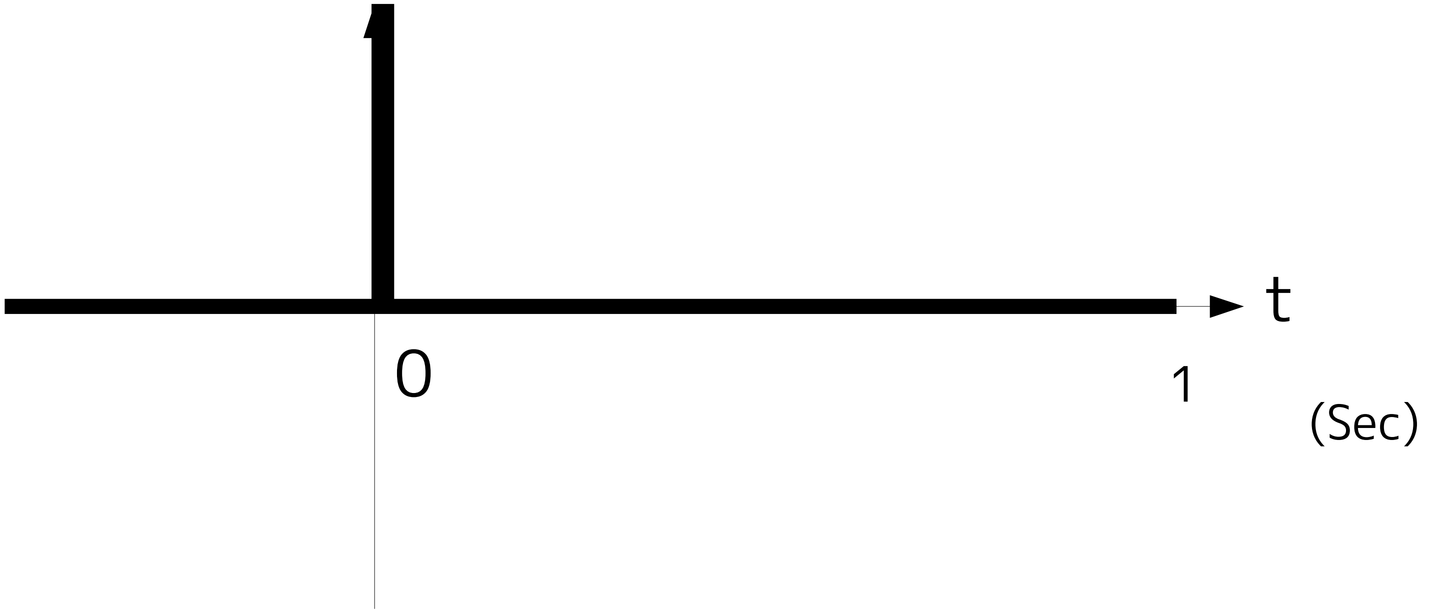
$$=\frac{1}{2\mathrm{i}}\int_{-\infty}^{\infty}(\mathrm{e}^{-i(2\pi f-2\pi)t}-\mathrm{e}^{-i(2\pi f+2\pi)t})dt$$

$$=\frac{1}{2\mathrm{i}}\left\{\int_{-\infty}^{\infty}e^{-i(2\pi f-2\pi)t}dt-\int_{-\infty}^{\infty}e^{-i(2\pi f+2\pi)t}dt\right\}$$

$$=\frac{i}{2}\left\{\delta(f+1)-\delta(f-1)\right\}$$

フーリエ変換の結果



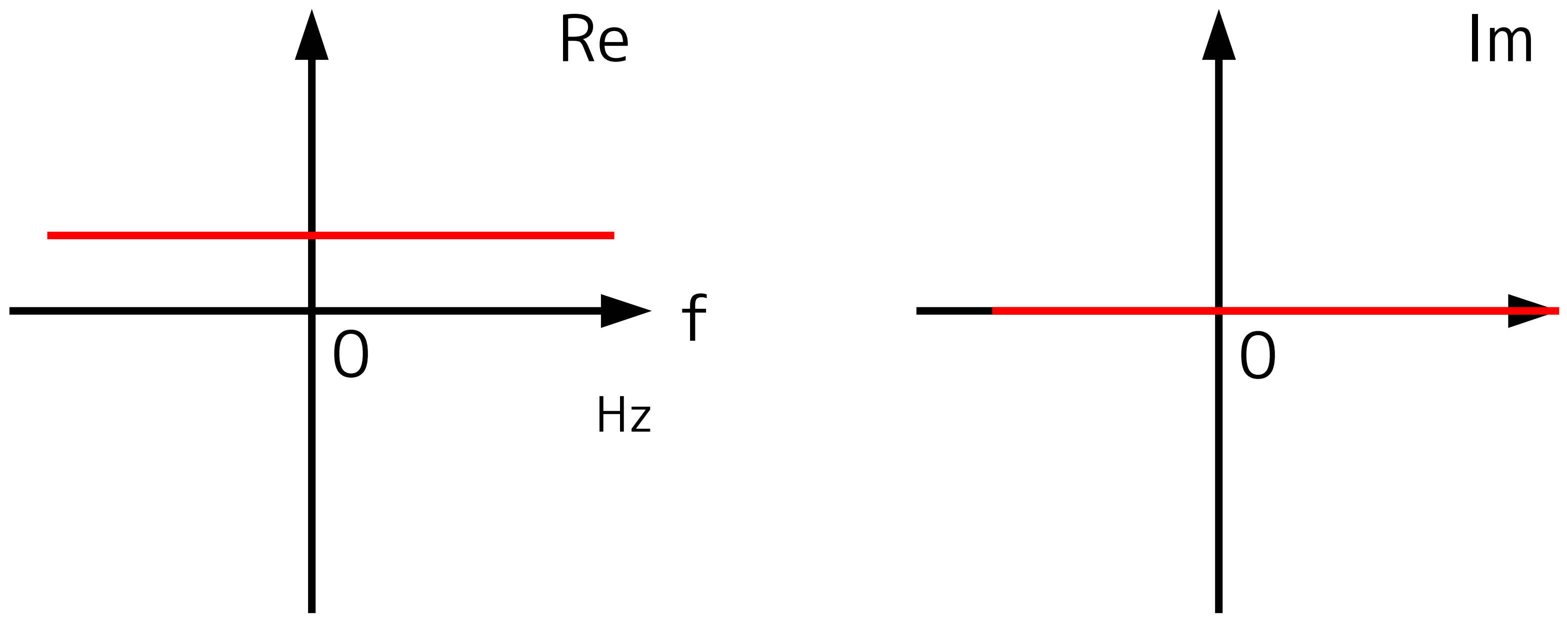

t
0
1
(Sec)

$$f(t)=\delta(t)$$

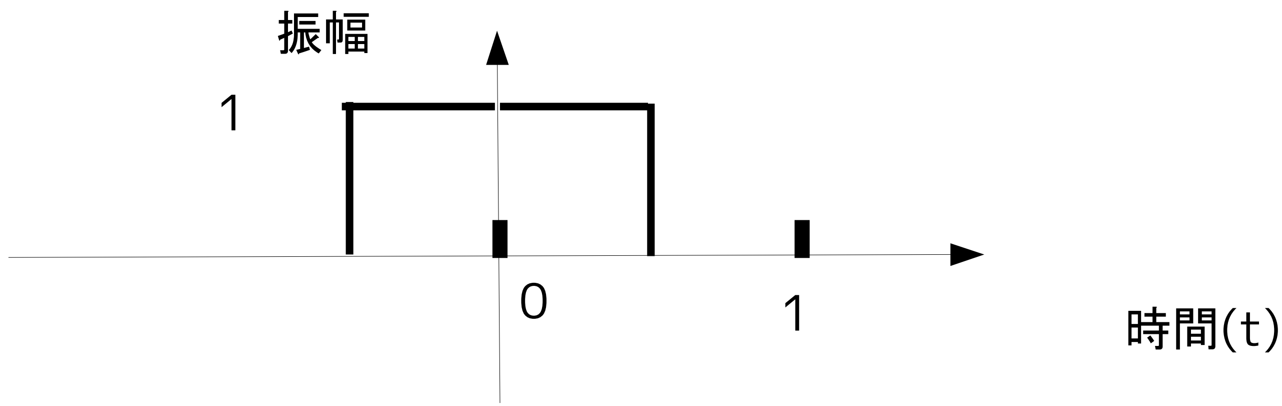
$$F(f)=\int_{-\infty}^{\infty}\delta(t)\,e^{-i2\pi f t}\,dt=\mathrm{e}^{0}=1$$

$$\int_{-\infty}^{\infty}\mathrm{x}(t)\,\delta(t-a)\,dt=x(a)\text{より}$$

フーリエ変換の結果



問題 以下の波形のフーリエ変換をもとめよ.



$$f(t)=\begin{cases}1 & \left(-\frac{1}{2}\leq t\leq \frac{1}{2}\right)\\ 0 & \end{cases}$$

フーリエ余弦級数より、

係数

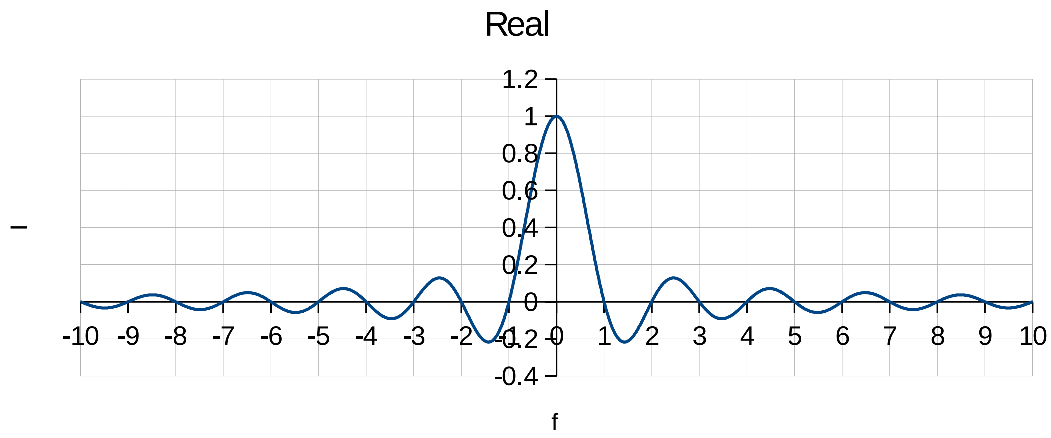
$$F(f)=\int_{-1/2}^{1/2} e^{-i2\pi f t}dt$$

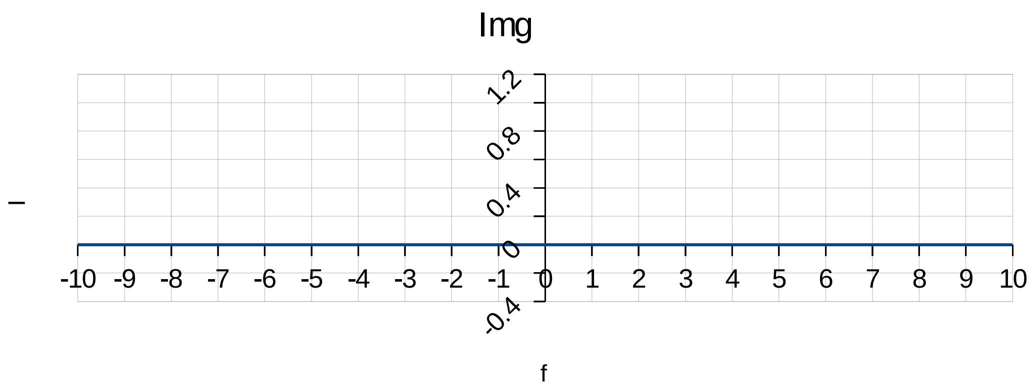
$$=\left[\frac{-1}{i2\pi f}e^{-i2\pi f t}\right]_{-1/2}^{1/2}$$

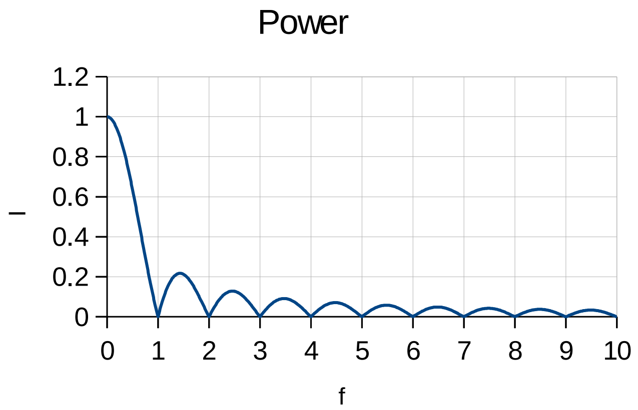
$$=\frac{1}{i2\pi f}\left(e^{i\pi f}-e^{-i\pi f}\right)$$

$$\frac{1}{(2i)}\left(e^{i\theta}-e^{-i\theta}\right)=\sin\theta \text{より}$$

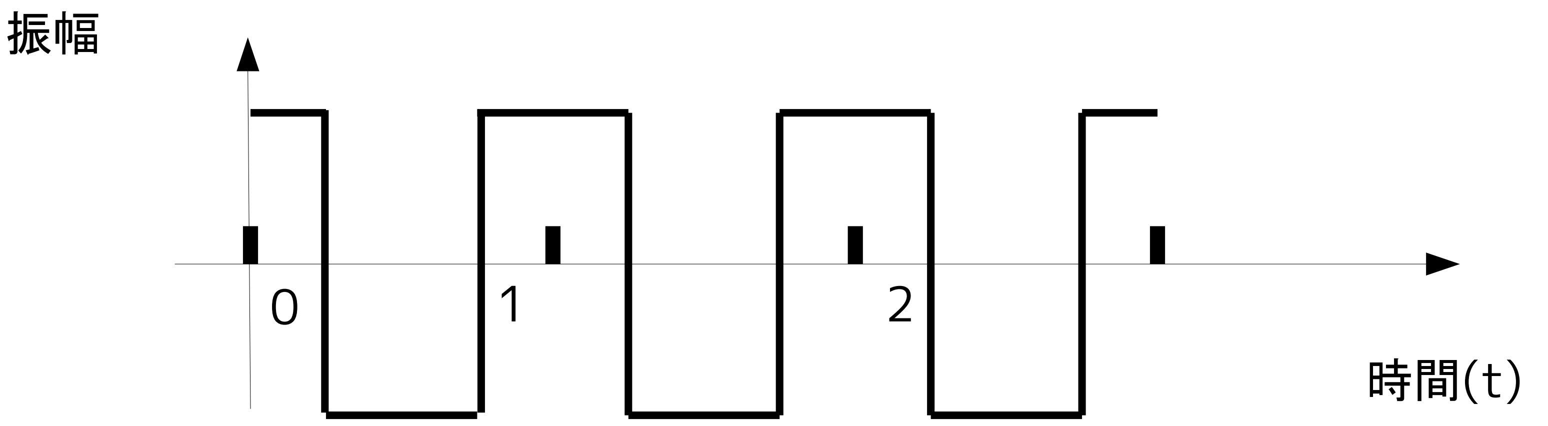
$$=\frac{\sin\pi f}{\pi f}$$

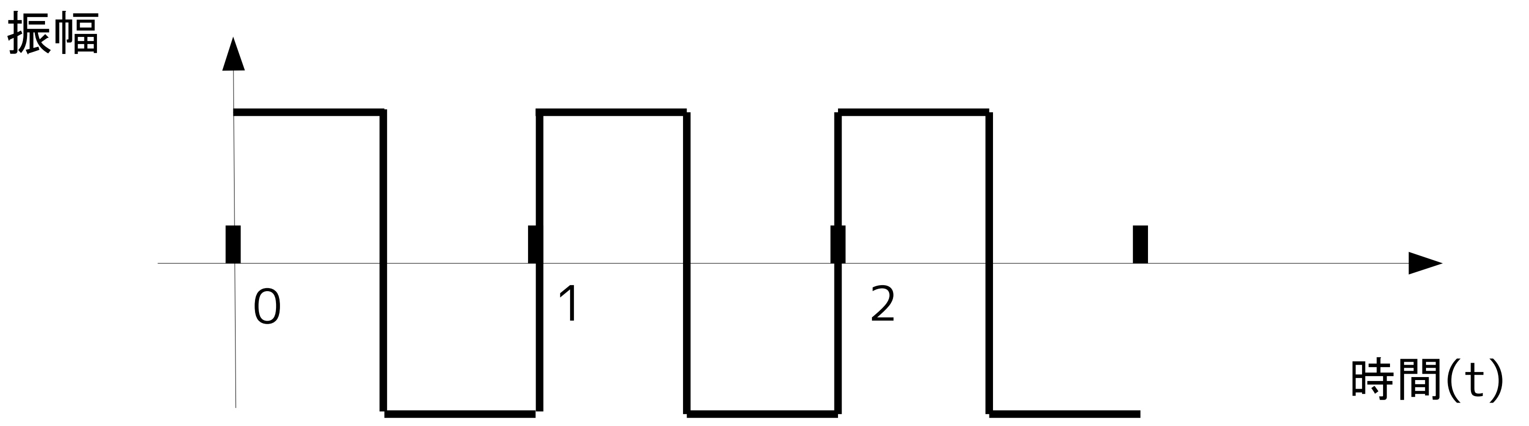

Real
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
f


Img
1.2
0.8
0.4
0
-0.4
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
f


Power
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
f

問題 以下の波形のフーリエ変換をもとめよ.





$$f(t)=\begin{cases} 1 & \left(-\frac{1}{2}\leqslant t< -\frac{1}{4}, \frac{1}{4}\leqslant t< \frac{1}{2}\right) \\ -1 & \left(-\frac{1}{4}\leqslant t< \frac{1}{4}\right) \end{cases}$$

フーリェ余弦級数より、

$$a_n=4\int_0^{\frac{1}{2}} f(t)\cos n\omega_0 t\,dt\,(n=1,2,3,\cdots)$$

$$=4\left\{\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_0^{\frac{1}{4}}-\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{\frac{1}{4}}^{\frac{1}{2}}\right\}$$

$\omega_0=2\pi$ より

$$=\frac{2}{n\pi}\left\{\left(\sin\frac{n\pi}{2}-0\right)-\left(0-\sin\frac{n\pi}{2}\right)\right\}$$

$$=\frac{4}{n\pi}\sin\frac{n\pi}{2}\quad(\text{このとき}a_0=0)$$

$$f(t)\sim\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\cos 2n\pi t$$

$$f(t)=\begin{cases}-1\left(-\frac{1}{2}\leqslant t<-\frac{1}{4},\frac{1}{4}\leqslant t<\frac{1}{2}\right)\\ 1\left(-\frac{1}{4}\leqslant t<\frac{1}{4}\right)\end{cases}$$

フーリエ余弦級数より、

係数 $$a_n=4\int_0^{\frac{1}{2}}f(t)\cos n\omega_0 t\,dt\,(n=1,2,3,\cdots)$$

$$=4\left\{\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_0^{\frac{1}{4}}-\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{\frac{1}{4}}^{\frac{1}{2}}\right\}$$ （$\omega_0=2\pi$ より）

$$=\frac{2}{n\pi}\left\{(\sin\frac{n\pi}{2}-0)-(0-\sin\frac{n\pi}{2})\right\}$$

$$=\frac{4}{n\pi}\sin\frac{n\pi}{2}$$ （このとき、 $a_0 \ni 0$

$$f(t)\sim\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\cos 2n\pi t$$

$$F(\omega)=\int_{-\infty}^{\infty}\left\{\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\cos n\omega_0 t\right\}e^{-j\omega t}dt$$

$$=\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\int_{-\infty}^{\infty}\cos n\omega_0 t\cdot e^{-j\omega t}dt$$

$$\left(\cos n\omega_0 t=\frac{e^{jn\omega_0 t}+e^{-jn\omega_0 t}}{2}\text{より}\right)$$

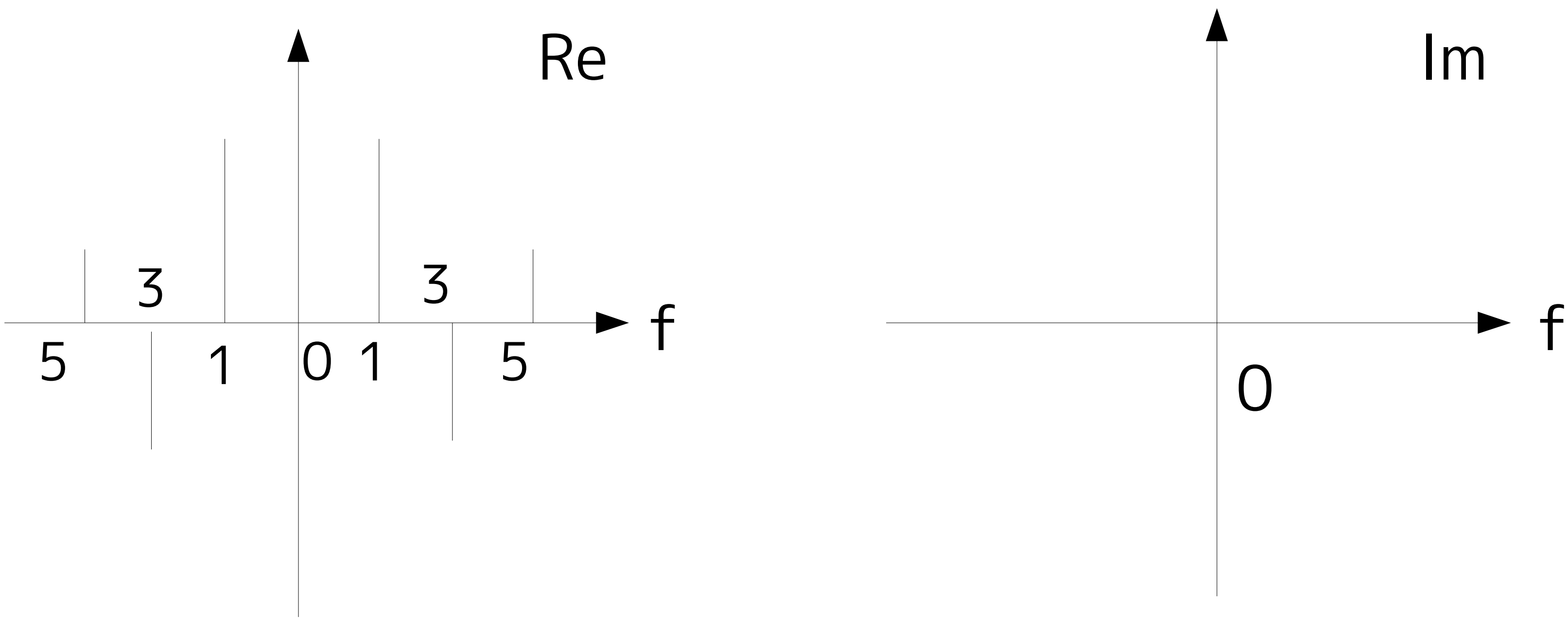
$$=\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\frac{1}{2}\int_{-\infty}^{\infty}\left(e^{-j(\omega-n\omega_0)t}+e^{-j(\omega+n\omega_0)t}\right)dt$$

$$=\frac{2}{\pi}\left\{\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\right\}\left(\delta(\omega-n\omega_0)-\delta(\omega+n\omega_0)\right)$$

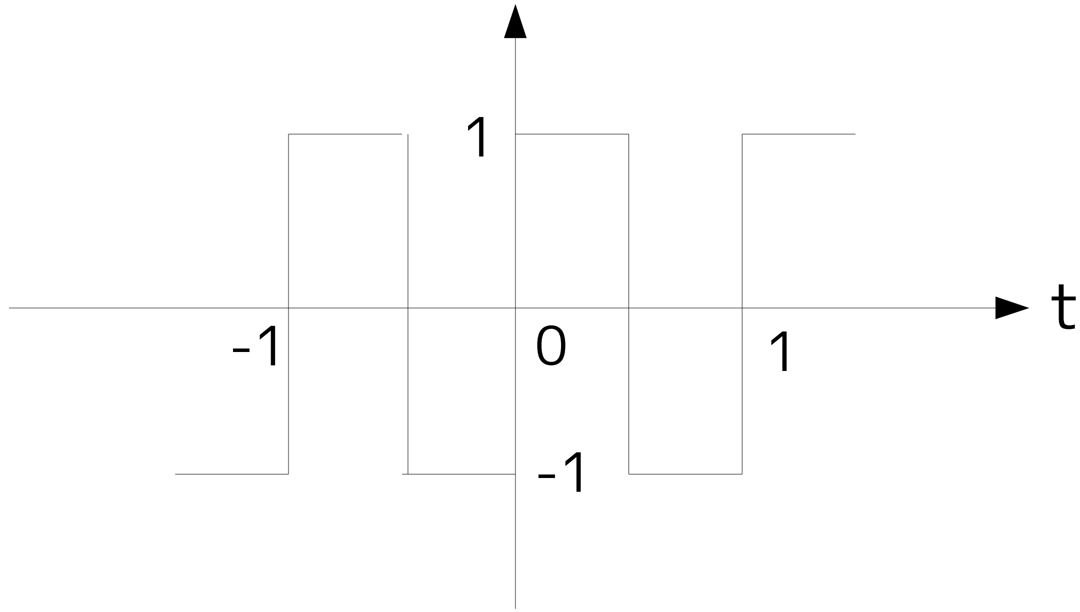
$$\left(\omega_0=2\pi\text{ より}\right)$$

$$=\frac{2}{\pi}\left\{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}\right\}\left(\delta(\omega-2n_0\pi)+\delta(\omega+2n_0\pi)\right)$$

$$(n_0=2n-1)$$



Re
3
3
f
5
1
0
1
5
Im
f
0


1
t
-1
0
1
-1

$$f(t)=\begin{cases} -1\left(-\frac{1}{2}\leqq t<0\right) \\ 1\left(\frac{1}{2}\leqq t<0\right) \end{cases}$$

フーリェ正弦級数より、

係数 $$b_n=4\int_0^{\frac{1}{2}} f(t)\sin n\omega_0 t dt\,(n=1,2,3,\cdots)$$

$$=4\left[\frac{-\cos n\omega_0 t}{n\omega_0}\right]_0^{\frac{1}{2}} \quad (\omega_0=2\pi より)$$

$$=\frac{2}{n\pi}(-\cos n\pi+1)$$

$$=\frac{2}{n\pi}(1-(-1)^n)$$

$$f(t)\sim\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}(1-(-1)^n)\right\}\sin 2n\pi t$$

$$F(\omega)=\int_{-\infty}^{\infty}\left\{\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(1-(-1)^{n}\right)\right\}\sin n\omega_0 t\right\}e^{-j\omega t}dt$$

$$=\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(1-(-1)^{n}\right)\right\}\int_{-\infty}^{\infty}\sin n\omega_0 t\cdot e^{-j\omega t}dt$$

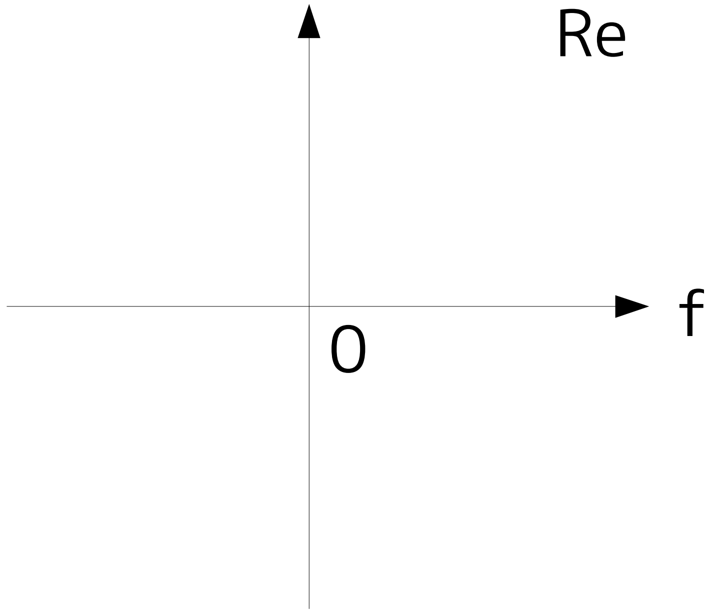
($\sin n\omega_0 t=\dfrac{e^{jn\omega_0 t}-e^{-jn\omega_0 t}}{2j}$ より)

$$=\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(1-(-1)^{n}\right)\right\}\frac{1}{2j}\int_{-\infty}^{\infty}\left(e^{-j(\omega-n\omega_0)t}-e^{-j(\omega+n\omega_0)t}\right)dt$$

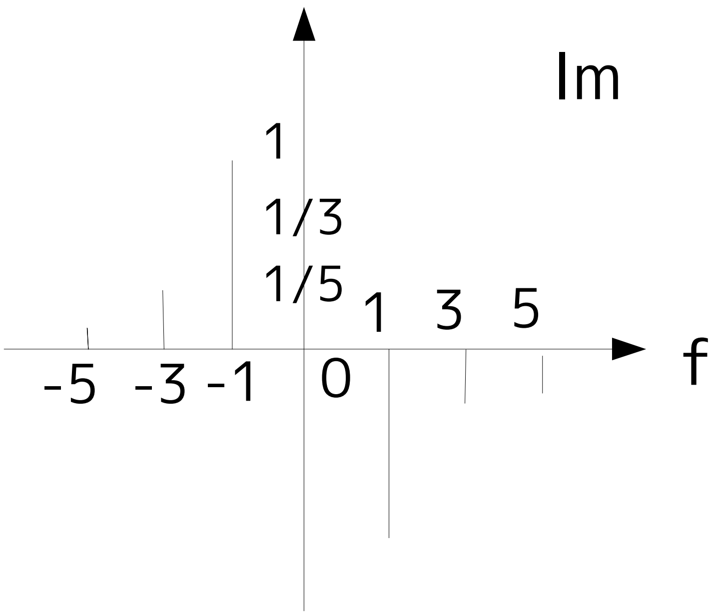
$$=\frac{1}{2j}\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(1-(-1)^{n}\right)\right\}\left(\delta(\omega-n\omega_0)-\delta(\omega+n\omega_0)\right)$$

($\omega_0=2\pi$ より)

$$=\frac{j}{2}\left\{\sum_{n=1}^{\infty}\frac{2}{n\pi}\left(1-(-1)^{n}\right)\right\}\left(\delta(\omega+2n\pi)-\delta(\omega-2n\pi)\right)$$



Re
0
f

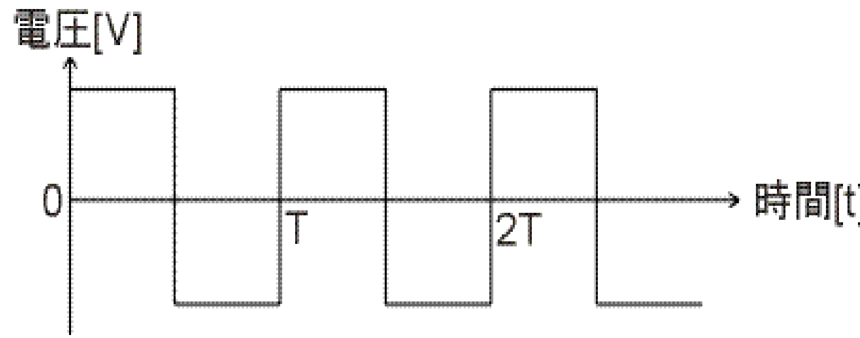


Im
1
1/3
1/5
1
3
5
f
-5
-3
-1
0

時間波形とフーリェ級数展開とフーリェ変換

方形波

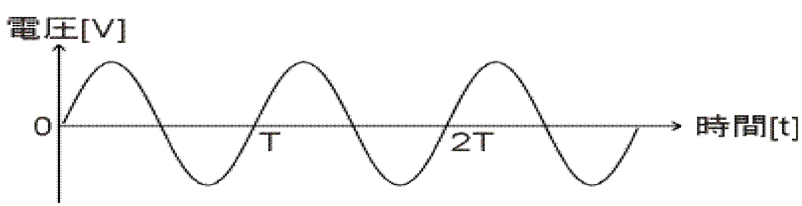
$R(t)$



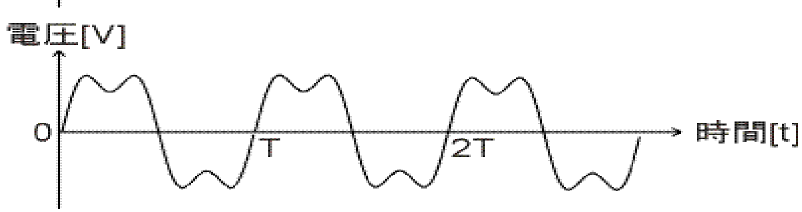
フーリェ級数展開

$$R(t)=\sin(2\pi t * f_0)+1/3*\sin(2\pi t*3f_0)+1/5*\sin(2\pi t*5f_0)+\dots$$

第１項



+第3項



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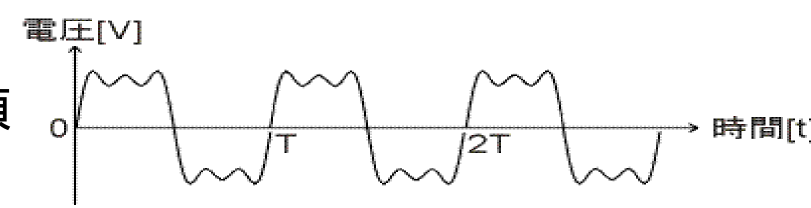
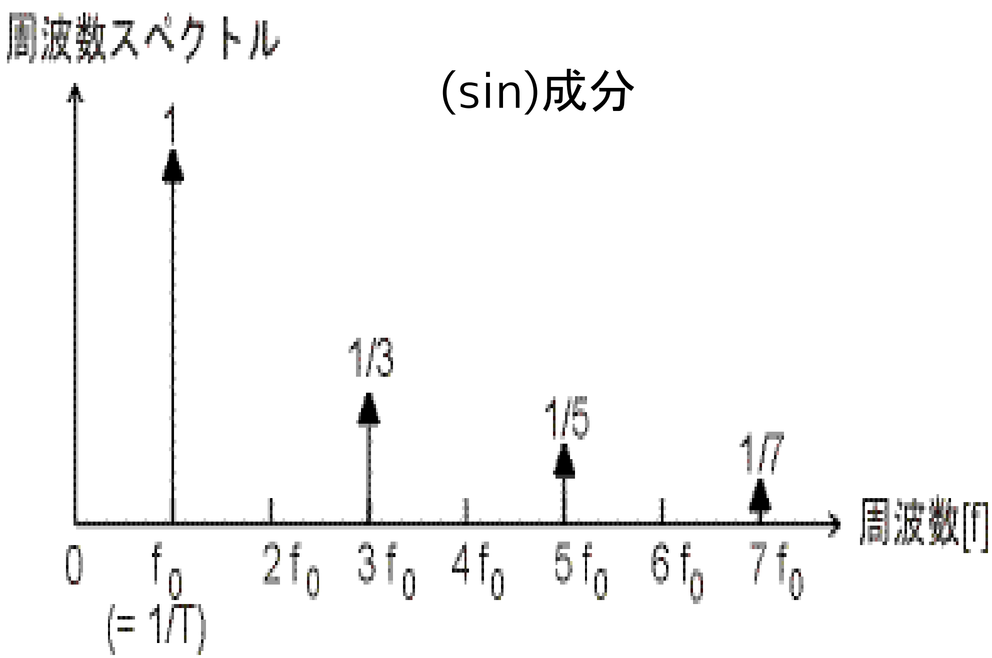


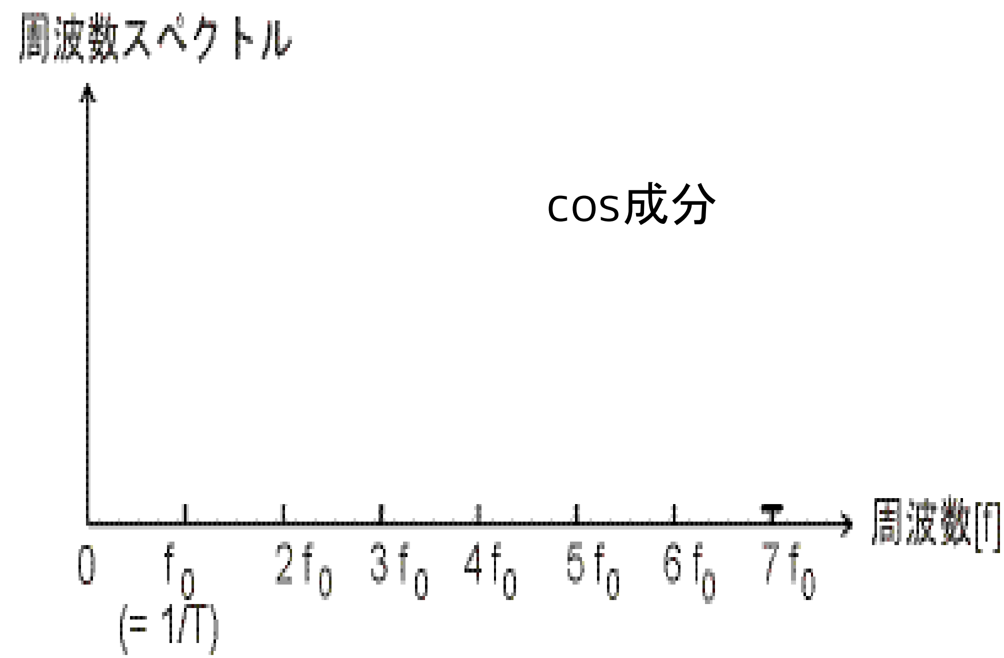
フーリェ級数展開

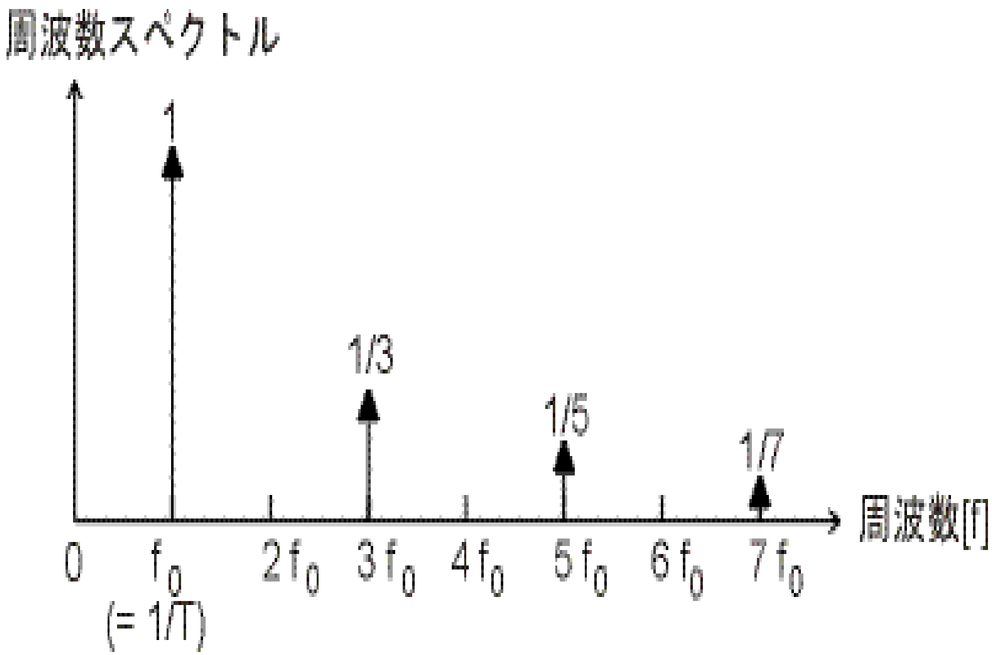
フーリェ変換の基本周波数成分のみ表示

$$R(t)=\sin(2\pi t*f_0)+1/3*\sin(2\pi t*3f_0)+1/5*\sin(2\pi t*5f_0)+\ldots$$

$$+0*\cos(2\pi t*f_0)+0*\cos(2\pi t*3f_0)+0*\cos(2\pi t*5f_0)+\ldots$$







（輝線スペクトル）